THERMAL CALCULATION OF ROTARY REGENERATORS WITH A DISPERSED PACKING

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The integral Laplace transformation and the reduction of differential equations to Volterra integral equations are used to obtain a solution to the equation of the packing and gas temperature distribution over the thickness of the section of a rotary radially sectioned regenerator with a dispersed packing in relation to time for the initial period of operation of the regenerator.

Rotary heat exchangers with a dispersed packing have several advantages over the regenerators presently used in industry. In this paper we present a theoretical thermal calculation of a rotary regenerative radially sectioned heat exchanger with a dispersed packing. A full description of the exchanger was given in [1].

As the diagram of the operation of the regenerator in [1] shows, this heat exchanger operates on the cross-flow principle. In fact, gas with a constant initial temperature encounters a packing with varying temperature, i.e., this process can be illustrated as shown in Fig. 1.

Heat transfer between the gas (air) and the packing is unsteady relative to each section of the packing, but in each cross section of the heat exchanger the temperature at any instant in steady-state operation is the same as it was at the same instant in the preceding cycle.

For this heat exchanger we have the following conditions of periodic stationarity:

$$\vartheta_1^{g}(\tau = 0) = \vartheta_2^{a}(\tau = \tau_a),$$
$$\vartheta_2^{g}(\tau = \tau_g) = \vartheta_1^{a}(\tau = 0). \qquad (1)$$

In the general case the external and internal problems must be considered in the investigation of heat transfer between the packing and the gas. In several cases, however, the temperature gradient over the cross section of the packing particles can be neglected. This is valid at particular values of Bi and Fo. In this paper we consider the case where the temperature gradient over the cross section of the particle can be neglected.

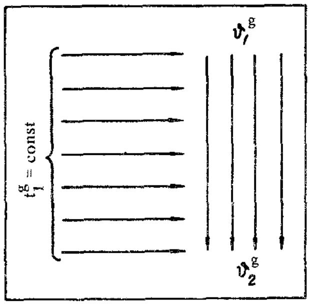

Fig. 1. Variation of temperatures of packing and gases in plane of entry of gases into section of packing (x = 0) in radially-sectioned regenerator.

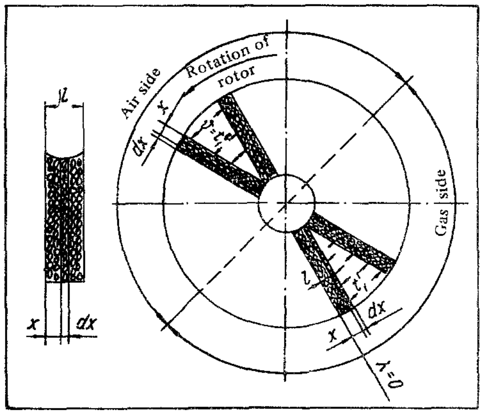

Fig. 2. For the deduction of differential equations for temperature of packing and gases (air).

The total heat flux from the gases to the packing and from the packing to the air involves convection, radiation, and conduction. In the case of point contact of the particles heat transfer between the particles by conduction can be neglected in certain conditions in comparison with the heat transfer from the gases to the packing [2, 3]; this case is also considered in this paper.

When convective heat transfer is intense, radiation at temperatures up to 500° C can also be neglected; at higher temperatures the effect of radiation can be taken into account by an over-all heat transfer coefficient:

$$\alpha = \alpha_c + \alpha_{\text{rad}}.$$

We will assume also that the temperature of the gases over the height of the apparatus is constant (as the diagram illustrating the operation of the apparatus indicates), and the filtration of gases through the layer of packing is uniform with the same velocity at any cross section x (Fig. 2).

In these conditions we can write the following equations for the hot side:

$$\alpha F (\vartheta^{g} - t^{g})\, dx d\tau = C_p L d\tau \frac{\partial t^{g}}{\partial x}\, dx, \qquad (2)$$

$$\alpha F (t^{g} - \vartheta^{g})\, dx d\tau = C_{\text{м}} G_{\text{м}} dx \frac{\partial \vartheta^{g}}{\partial \tau}\, d\tau. \qquad (3)$$

We introduce the dimensionless variables

$$\lambda = \frac{\alpha F}{C_p L} x, \tag{4}$$

$$\delta = \frac{\alpha F}{C_{\text{м}} G_{\text{м}}} \tau. \tag{5}$$

Then from (2) and (3) we obtain

$$\frac{\partial t^g}{\partial \lambda} = \vartheta^g - t^g, \tag{6}$$

$$\frac{\partial \vartheta^g}{\partial \delta} = t^g - \vartheta^g. \tag{7}$$

We put

$$\Theta^g = (t_{\mathrm{I}}^g - \vartheta^g)/(t_{\mathrm{I}}^g - \vartheta_1). \tag{8}$$

From Eqs. (6) and (7) with reference to (8) we obtain the following differential equation for the packing temperature in any gas period:

$$\frac{\partial^2 \Theta^g}{\partial \lambda \partial \delta} + \frac{\partial \Theta^g}{\partial \lambda} + \frac{\partial \Theta^g}{\partial \delta} = 0. \tag{9}$$

We will formulate the boundary conditions. For an infinitely narrow strip of heat-transfer surface located on the side of entry of the gases into each section of the packing, the gas temperature is constant in time, i.e., the water equivalent W_g of the gases is infinitely large. When one of the heat-transfer media has a constant temperature the equation representing the change in temperature of the second medium (in this case the packing material) for the first gas period can be obtained by substituting the value $W_g = \infty$ in the theoretical heat-transfer equations [4]. Hence,

$$\vartheta_{\mathrm{I}}^g(0, \delta) = t_{\mathrm{I}}^g - (t_{\mathrm{I}}^g - \vartheta_1) \exp\left(-\frac{\alpha F'}{W_{\text{м}}}\right) \tag{10}$$

or, with reference to (5), condition (10) takes the form

$$\vartheta_1(0, \delta) = t_{\mathrm{I}}^g - \Delta t_{\mathrm{I}}^g \exp(-\delta), \tag{11}$$

where $\Delta t_{\mathrm{I}}^g = t_{\mathrm{I}}^g - \vartheta_1$. Hence, for the dimensionless temperature (8) in the first gas period we have

$$\Theta_{\mathrm{I}}^g(0, \delta) = \exp(-\delta). \tag{12}$$

To determine the initial conditions we consider the startup period of the apparatus. Before the onset of the first gas period the cold (air) side of the packing has a constant temperature ϑ_1 with which it approaches the hot side (we assume here that the initial packing temperature ϑ_1 is equal to the temperature t_1^a of the ambient air), i.e., we have the following initial condition for the first gas period:

$$\vartheta(\lambda, 0) = \vartheta_1,$$

or

$$\Theta_{\mathrm{I}}^g(\lambda, 0) = 1. \tag{13}$$

We solve Eq. (9) by using the integral Laplace transformation for the variable λ.

The transformation for λ is perfectly valid in this case (the temperature of the gases in the space between the sections is constant) since the packing temperature in any section is independent of the finite length of the packing in the x direction.

In the image region we obtain the following equation in place of Eq. (9):

$$\frac{d}{d\delta}[s\overline{\Theta}_{\mathrm{I}}^g(s, \delta) - \Theta_{\mathrm{I}}^g(0, \delta)] +$$

$$+ s\overline{\Theta}_{\mathrm{I}}^g(s, \delta) - \Theta_{\mathrm{I}}^g(0, \delta) + \frac{d\overline{\Theta}_{\mathrm{I}}^g(s, \delta)}{d\delta} = 0. \tag{14}$$

With the boundary conditions (12) Eq. (14) after appropriate transformations for the first gas period takes the form

$$\frac{d\overline{\Theta}_{\mathrm{I}}^g(s, \delta)}{\overline{\Theta}_{\mathrm{I}}^g(s, \delta)} = -\frac{s}{s+1} d\delta, \tag{15}$$

the solution of which is

$$\overline{\Theta}_{\mathrm{I}}^g(s, \delta) = C(s) \exp\left(-\frac{s}{s+1}\delta\right). \tag{16}$$

To determine the constant of integration C(s) we use the initial condition (13). Then, from Eq. (16) with $\delta = 0$, we obtain

$$C(s) = 1/s.$$

Substituting this value of C(s) in Eq. (16) we obtain the solution of Eq. (15):

$$\overline{\Theta}_{\mathrm{I}}^g(s, \delta) = \frac{1}{s} \exp\left(-\frac{s}{s+1}\delta\right). \tag{17}$$

To find the original of the function $\overline{\Theta}_{\mathrm{I}}^g(s, \delta)$ we expand the right side of Eq. (17) in a series:

$$\frac{1}{s}\exp\left(-\frac{s}{s+1}\delta\right) = \frac{1}{s} - \frac{\delta}{1!}\frac{1}{s+1} +$$

$$+ \frac{\delta^2}{2!}\frac{s}{(s+1)^2} - \frac{\delta^3}{3!}\frac{s^2}{(s+1)^3} +$$

$$+ \frac{\delta^4}{4!}\frac{s^3}{(s+1)^4} - \ldots = \sum_{k=0}^{\infty} \frac{(-\delta)^k}{k!}\frac{s^{k-1}}{(s+1)^k}. \tag{18}$$

We determine the originals of the first four terms of series (18) from the tables in [5].

We determine the originals of the remaining terms of the series by expansion of each of these terms in series,

$$\sum_{k=0}^{\infty} \frac{s^{k-1}}{(s+1)^k} = \frac{A}{s+1} + \frac{B}{(s+1)^2} +$$

$$+ \frac{C}{(s+1)^3} + \ldots + \frac{M}{(s+1)^{k-1}} + \frac{N}{(s+1)^k}, \tag{19}$$

where the coefficients A, B, C, ... M, N are determined by known methods.

The originals of each term of series (19) are determined from the formula [5]:

$$\frac{1}{(s+1)^n} \doteqdot \frac{\lambda^{n-1}}{(n-1)!} \exp(-\lambda).$$

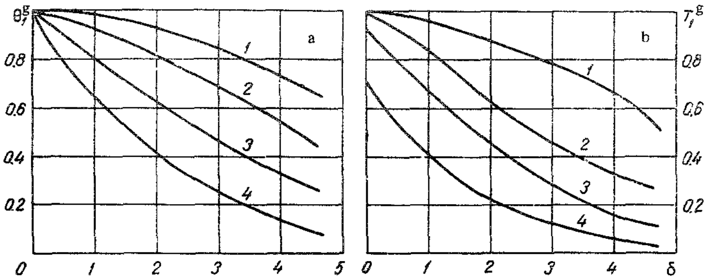

Fig. 3. Change in dimensionless temperatures of packing (a) and gases (b), calculated from equations (27) and (29), in relation to time for different λ and δ: 1) λ = 6.15; 2) λ = 3.7; 3) λ = 2.46; 4) λ = 1.23.

Then the original $\Theta_1^g(s, \delta)$ will be

$$\Theta_1^g(s,\ \delta) = 1 - \left[\delta - \frac{\delta^2}{2!}(1-\lambda) + \right.$$
$$+ \frac{\delta^3}{3!}\left(1 - 2\lambda + \frac{\lambda^2}{2}\right) - \frac{\delta^4}{4!}\left(-\frac{\lambda^3}{3!} + \frac{3}{2}\lambda^2 - \right.$$
$$\left. - 3\lambda + 1\right) + \frac{\delta^5}{5!}\left(\frac{\lambda^4}{4!} - \frac{2}{3}\lambda^3 + \right.$$
$$\left. \left. + 3\lambda^2 - 4\lambda - 1\right) - \ldots \right] \exp(-\lambda). \tag{20}$$

It is easy to see that Eq. (20) satisfies the boundary conditions (12) and (13).

To find the packing temperature in subsequent periods of operation of the regenerator the method of integral Laplace transformations is most suitable. However, the original of the image (17), found in the form of expression (20), is awkward and this makes the subsequent transformations difficult. The original of the image (17) can be found from the solution of Eq. (9) by reducing the differential equation to an integral equation. As will be shown below, this solution is simpler.

By the substitution

$$\Theta_1^g(\lambda,\ \delta) = W(\lambda,\ \delta)\exp(-\lambda-\delta) + \exp(-\delta) \tag{21}$$

Eq. (9) is brought to the form

$$\frac{\partial^2 W(\lambda,\ \delta)}{\partial\lambda\,\partial\delta} - W(\lambda,\ \delta) = \exp(\lambda) \tag{22}$$

with boundary conditions obtained from (12) and (13) with due regard to (21):

$$W(0,\ \delta) = 0, \tag{23}$$

$$W(\lambda,\ 0) = 0. \tag{24}$$

Integrating Eq. (22) with respect to λ and δ, after some transformations using (23) and (24) we obtain the following Volterra integral equation of the second kind:

$$W(\lambda,\ \delta) = \delta\,[\exp(\lambda) - 1] + \int_0^\delta\int_0^\lambda W(\xi,\eta)\,d\xi\,d\eta, \tag{25}$$

the solution of which is

$$W(\lambda,\ \delta) = \sum_{k=1}^{\infty}\frac{\delta^k}{k!}\left[\exp(\lambda) - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right]. \tag{26}$$

Converting to the variable $\Theta_1^g(\lambda, \delta)$ from relationship (21), we finally obtain an expression for the dimensionless temperature of the packing in the first gas period:

$$\Theta_1^g(\lambda,\ \delta) = \exp(-\delta) + \exp(-\lambda-\delta)\times$$
$$\times\sum_{k=1}^{\infty}\frac{\delta^k}{k!}\left[\exp(\lambda) - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right]. \tag{27}$$

Series (27) is absolutely convergent and converges fairly rapidly.

It is easy to see that Eq. (27) satisfies Eq. (9) and boundary conditions (12) and (13). We can show that series (20) can be replaced by series (21). In fact, from the theorem of uniqueness of the solution of the mixed problem, series (20) and (27), each of which is the solution of Eq. (9) with boundary conditions (12) and (13), are identical. Hence, to find the temperatures in the subsequent periods we use the relationship

$$\frac{1}{s}\exp\left(-\frac{s}{s+1}\delta\right) \risingdotseq \exp(-\delta) + \exp(-\lambda-\delta)\times$$
$$\times\sum_{k=1}^{\infty}\frac{\delta^k}{k!}\left[\exp(\lambda) - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right]. \tag{28}$$

To find the packing temperatures in subsequent periods by the reduction of the differential equation to an integral one is too awkward and, hence, in subsequent calculations we resort to a combination of the two methods by means of expression (28).

Substituting (27) in Eq. (7) and referring to (8) we obtain an expression for the dimensionless temperature of the gases in the first gas period:

$$T_1^g(\lambda,\ \delta) = \exp(-\lambda-\delta)\times$$
$$\times\sum_{k=1}^{\infty}\frac{\delta^{k-1}}{(k-1)!}\left[\exp(\lambda) - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right], \tag{29}$$

where

$$T_1^g = [t_1^g - t^g(\lambda,\ \delta)]/(t_1^g - \vartheta_1). \tag{30}$$

Series (29) is also absolutely convergent.

Figure 3 shows that with increase in λ and δ = const the values of Θ_1^g and T_1^g decrease. When δ becomes infinitely large functions Θ_1^g and T_1^g tend to zero. The calculations from formulas (27) and (29) were performed on a Minsk-2 electronic digital computer.

NOTATION

C_p is the specific heat of gases, J/kg · deg; C_M is the specific heat of packing material, J/kg · dg; F is the surface of particles per m thickness of section of packing, m^2/m; F' is the surface of particles, m^2; G_M is the weight of packing per m thickness of section of packing, kg/m; L is the mass flow rate of gas passing through one section of packing, kg/h; l is the total thickness of section of packing, m; t^g is the instantaneous temperature of gases passing through section of packing in gas period, °K; t_1 is the initial temperature of gas (in gaps between sections of packing), °K; W_M is the water equivalent of packing material, W/deg; x is the variable coordinate, m; α is the heat transfer coefficient, $W/m^2 \cdot deg$; ϑ° is the packing temperature at start of first gas period; ϑ_1^g is the packing temperature at start of any gas period; ϑ_1^a is the packing temperature at start of any air period; ϑ_2^g is the packing temperature at end of any gas period; ϑ_2^α is the packing temperature at end of any air period; τ is the time, hr; τ_g is the total duration of gas period, hr; ϑ^g is the instantaneous packing temperature in gas period, °K.

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